**Advanced Algorithms**

**Exercise for Lecture 14**

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| **Student Name** |  | **Student ID** |  |
| **Problem 1** |  | | |
| **Problem 2** |  | | |
| **Problem 3** |  | | |
| **Total Score** |  | | |
| **Notes** | Deadline: **2023-11-12 24:00**  Submission Format: ‘**Lecture14\_Name\_Student ID.docx**’, and please send to: **[algorithms\_23fall@163.com](mailto:algorithms_23fall@163.com)**.  This assignment is meant to be an evaluation of your **individual** understanding coming into the course and should be completed **without collaboration** or outside help. | | |

**Problem 1. [30 points]** Suppose that you want to multiply the two polynomials and using the DFT. Choose an appropriate power of two, find the DFT of the two sequences, multiply the results component-wise, and compute the inverse DFT to get the final result. (Hint: , where the entry of is ).

**Solution:**

Multiplication of a polynomial of degree with a polynomial of degree yields a polynomial of degree . Obviously, , and is a power of . We have . [5 points]

The DFT of the first polynomial is [5 points]

.

For the second polynomial we have [5 points]

.

The coefficients of the product polynomial are now given by the inverse DFT of the point-wise product of these value representations. That is, [10 points]

.

So, the product polynomial is . [5 points]

**Problem 2. [30 points]** Show how FFT computes the DFT of the vector .

**Solution:**

Where ,

.

Here, , . [10 points]

Set , .

Here, , .

Then, , , . [5 points]

If , and are DFT with points.

So, we have , , , .

Thus, , , , . [10 points]

So, (where ) are DFT of , it’s . [5 points]

**Problem 3. [40 points]** Show how FFT computes the DFT of the vector .

**Solution:**

Where ,

.

Here, , . [5 points]

Set , .

Here, , .

Then, , , .

If , and are DFT with points. [5 points]

.

Here, , .

Set , .

Then, , , .

Similarly, set , .

Then, , , . [10 points]

, , and all are DFT with points.

So, we have , , , , , , , .

Then, we have , , , ; , , , . [10 points]

Thus, , , , , , , , .

So, (where ) are DFT of , it’s . [10 points]